Students must demonstrate an in-depth analysis of a question or problem that has a mathematical focus. This investigation might be, for example:

- the applicability of mathematics to solve both real and abstract problems

- the beauty of mathematics, as in, for instance, geometry or fractal theory

- the elegance of mathematics in the proving of theorems as in, for example, number theory

- the origin and subsequent development of a branch of mathematics over a period of time, measured in tens, hundreds or thousands of years

- the links between different branches of mathematics, or the way that branch of mathematics has been born, or has flourished, as a result of technology.

Students should be advised on the importance of formulating logical and coherent reasons for selecting a particular topic for the EE, the need to identify a well-thought-out research question and the requirement to search for the mathematical problems that require a solution.

Students must be advised that mathematical research is a long-term and open-ended exploration of a set of related mathematical problems that are based on personal observations. The answers to these problems connect to and build upon each other over time.

The process of topic selection

Mathematics is the study of patterns, quantities, numbers, shapes and space using logical processes, rules and symbols. Students are therefore encouraged to investigate patterns, formulate new conjectures and determine truth by drawing conclusions from axioms and definitions in the process of selecting their topic.

Students should initially identify the broad area of inquiry that they are interested in. They are encouraged to pursue their interest in mathematical topics from other relevant fields such as engineering, the sciences or the social sciences.

Students then need to narrow down their topic by dividing the area into more specific and detailed subtopics.

For instance, a student might be interested in conducting research on geometry and its usage in navigation. They could narrow down the topic by focusing on "The role of mathematics, and geometry in particular, in navigation when sailors relied on the stars. Does it still play a part now that sailors have access to constructed satellites?"

After a careful analysis of a topic and its selection, a student may start working on designing a coherently written paper.

Often, their previous experiences help students to decide on their topic.

For some, a topic that extends beyond work already undertaken as part of the course might be a great source of research ideas. Students can extend their knowledge beyond the mathematics course covered in the Diploma Programme, or apply techniques used in their mathematics course to model in an appropriately selected topic.

Another source of inspiration might be the works on mathematical problems and theorems that are published in academic journals. Students can refer to these types of publications, in addition to searching e-resources, databases and reference guides on mathematical proof, investigative nature and mathematical modelling.

Students should use specialized academic search engines as results from standard search engines are not always appropriate for citation in a research paper.

Other excellent sources of inspiration include: unpublished conference papers, previously published essays and book chapters or journal articles published on reputable scholarly websites. When searching through the e-resources, students should remember that previously published papers can be located by a given author, topic, certain word or phrase found in titles or reviews. A school librarian is well placed to give advice on this.

Literature review—demonstrating knowledge and understanding in context

Literature-based research is an essential element of the EE. Students should review the existing literature on their topic to inform the construction of their own research questions and design.

While a literature review for mathematics might not be as extensive as in other subjects, students are expected to demonstrate their knowledge and understanding of the mathematics they are using in the wider context of the broader discipline—for example, how it has been applied before, or in another area.

Typically, determining one excellent source of information relevant to the students' research launches a chain reaction as some of the references in that article or book may also be relevant to the topic area.

This literature review should be conducted in the early stages of the research process, and could form the basis of an annotated bibliography, for example. A comprehensive literature review will help to guide and improve the students' own work as it will enable them to contextualize their findings.

After students have discussed their choice of topic with their supervisor, they can start to outline the main points to be discussed in their essay.

Research question

When working on the research question, students should be guided by the rationale that their mathematical investigation is interesting and that it fills a gap in their personal curiosity.

The research question should therefore be non-trivial, worthy of investigation and follow from the existing body of literature on the topic. The research question needs to be specific and sharply focused, be stated clearly on the title page, in the introduction of the essay, and be appropriately related to the discipline of mathematics. Students need to avoid researching questions that are too narrow or too obvious as this will limit their ability to formulate reasoned arguments.

A well-constructed research question should be expressed as a question, be specific, address an important and relevant issue in the field and try to offer an alternative perspective compared to previous research findings. In particular, a research question in mathematics should revolve around the application of good mathematical practice to the selected topic.

Research methods

Students should be guided by analysis of primary and secondary sources in conducting research. Students should be advised to analyse data using appropriate techniques, to reason arguments correctly, to model situations using correct methodology, to state problems clearly and to apply techniques at the correct level of sophistication in order to find a solution to the problems posed.

There must be sufficient explanation and commentary throughout the essay to ensure that the reader does not lose sight of the purpose of the essay in a mass of mathematical symbols, formulas and analysis. A good way to do this is to refer back to the research question and focus on the purpose of the essay as outlined in the introductory sections.

A primary source of research in mathematics involves data-gathering, visualization, abstraction, conjecturing and proof. Throughout the EE students should communicate mathematically:

- describing their way of thinking

- writing definitions and conjectures

- using symbols, theorems, graphs and diagrams

- justifying their conclusions.

A secondary source of research refers to a comprehensive review of scholarly work, including books, journal articles, essays in an edited book collection, and mathematical techniques incorporated in the publication that is the focus of the student's research.

Supervisors need to ensure that students are aware of their responsibility to cite properly the resources used and check their work for plagiarism. Citations should adhere to the requirements of the IB and be correctly and consistently applied.

Framework for the EE in mathematics

Introduction	An EE in mathematics is intended for students who are writing on any topic that has a mathematical focus and it need not be confined to the theory of mathematics itself. Mathematics essays are divided into six categories: • the applicability of mathematics to solve both real and abstract problems • the beauty of mathematics as in, for instance, geometry or fractal theory • the elegance of mathematics in the proving of theorems • the origin and subsequent development of a branch of mathematics over a period of time • the links between different branches of mathematics and the powerful structures that enable many seemingly different problems to be solved by a single theory • the way that a branch of mathematics has been born, or has flourished, as a result of technology.
Methods most relevant to subjects in this group	A primary source of research in mathematics involves data-gathering, visualization, abstraction, conjecturing and proof. Throughout the EE students should communicate mathematically: describing their way of thinking; writing definitions and conjectures; using symbols, theorems, graphs and diagrams; justifying their conclusions; and reading mathematics. A secondary source refers to a scholarly work, including books, journal articles, essays in an edited book collection, or reviews, and mathematical techniques incorporated in the publication that is the focus of student's research.
Suggestions for possible sources	Use of peer-reviewed journals, personal conversation with the teacher, books, e-resources and publications online, personal investigation into the topic, specialized academic research engines, unpublished conference papers, previously published essays and book chapters, and single author books, among other sources.

Particular things to be aware of	Students need to be aware that their work will be checked in terms of the IB's academic honesty policy and so all students must ensure that they are familiar with this document.
Summary	Undertaking an EE is a challenge and so planning is crucial. Students need to remember to start writing their papers early and discuss any emerging difficulties with their supervisor. Supervisors and librarians are a great source of information, advice and support for students. Students writing a mathematics EE should search for primary and secondary sources of information prior to initiating the writing process. The framing of a good and robust research question, which is focused, will aid students in establishing a reasoned argument.
The EE and internal assessments	The EE is not an extension of the internal assessment and students must ensure that they are not using material submitted for any other assessment component as part of the EE submission—see subject-specific guidance for more details.